

# SPACECRAFT DOPPLER TRACKING AS A XYLOPHONE DETECTOR

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## Abstract

*We discuss spacecraft Doppler tracking in which Doppler data recorded on the ground are linearly combined with Doppler measurements made on board a spacecraft. By using the four-link radio system first proposed by Vessot and Levine<sup>[1]</sup>, we derive a new method for removing from the combined data the frequency fluctuations due to the Earth troposphere, ionosphere, and mechanical vibrations of the antenna on the ground. Our method provides also a way for reducing by several orders of magnitude, at selected Fourier components, the frequency fluctuations due to other noise sources, such as the clock on board the spacecraft or the antenna and buffeting of the probe by non gravitational forces<sup>[2]</sup>. In this respect spacecraft Doppler tracking can be regarded as a xylophone detector.*

*Estimates of the sensitivities achievable by this xylophone are presented for two tests of Einstein's theory of relativity: searches for gravitational waves and measurements of the gravitational red shift.*

*This experimental technique could be extended to other tests of the theory of relativity, and to radio science experiments that rely on high-precision Doppler measurements.*

## INTRODUCTION

Spacecraft Doppler tracking is the most sensitive technique to date for measuring distances and velocities of objects in the solar system, leading to information on masses and higher order moments of gravity fields of planets, their satellites, and asteroids<sup>[3, 4]</sup>. Doppler measurements have also been utilized to search for gravitational waves in the millihertz frequency region<sup>[5, 6]</sup>, and for placing upper limits on amplitudes of signals characterizing relativistic effects<sup>[7, 8]</sup>. These Doppler observations, however, suffer from noise sources that can be, at best, partially reduced or calibrated by implementing specialized and expensive hardware. The fundamental limitation is imposed by the frequency fluctuations inherent in the clocks referencing the microwave system. Current generation hydrogen masers achieve their best performance at about 1000 seconds integration time with a fractional frequency stability of a few parts in  $10^{-16}$ . This integration time is also comparable to the propagation time to spacecraft in the outer solar system.

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The frequency fluctuations induced by the intervening media have severely limited the sensitivities of these experiments. Among all the propagation noise sources, the troposphere is the largest and the hardest to calibrate to a reasonably low level. Its frequency fluctuations have been estimated to be as large as  $10^{-13}$  at 1000 seconds integration time<sup>[9]</sup>.

In order to systematically remove the frequency fluctuations due to the troposphere in the Doppler data, it was pointed out by Vessot and Levine<sup>[1]</sup> and Smarr *et al.*<sup>[10]</sup> that by adding to the spacecraft payload a highly stable frequency standard, a Doppler readout system, and by utilizing a transponder at the ground antenna, one could make Doppler one-way (earth-to-spacecraft, spacecraft-to-earth) as well as two-way (spacecraft-earth-spacecraft, earth-spacecraft-Earth) measurements. This way of operation makes the Doppler link totally symmetric and allows the complete removal of the frequency fluctuations due to the earth troposphere, ionosphere, and mechanical vibrations of the ground antenna by properly combining the Doppler data recorded on the ground with the data measured on the spacecraft. Their proposed scheme relied on the possibility of flying a hydrogen maser on a dedicated mission. Although current designs of hydrogen masers have demanding requirements in mass and power consumption, it seems very likely that by the beginning of the next century new space-qualified atomic clocks, with frequency stability of a few parts in  $10^{-16}$  at 1000 seconds integration time, will be available. They would provide a sensitivity gain of almost a factor of one thousand with respect to the best performance crystal-driven oscillators. Although this clearly would imply a great improvement in the technology of spaceborne clocks, it would not allow us to reach a Doppler sensitivity better than a few parts  $10^{-16}$ . This would be only a factor of five or ten better than the Doppler sensitivity expected to be achieved on the future Cassini project, a NASA mission to Saturn, which will take advantage of a high radio frequency link (32 GHz) in order to minimize the plasma noise, and will use a specially built water vapor radiometer for calibrating up to 95% of the frequency fluctuations due to the troposphere<sup>[11]</sup>.

In this paper we adopt the radio link configuration first envisioned by Vessot and Levine<sup>[1]</sup>, but we combine the Doppler responses measured on board the spacecraft and on the ground in a different way, as it will be shown in the following sections. Furthermore our technique allows us to reduce by several orders of magnitude, at selected Fourier components, the noise due to the clock on board the spacecraft, or to the antenna and buffeting of the probe by non gravitational forces<sup>[2]</sup>. This experimental approach could also be extended to other tests of the relativistic theory of gravity and to radio science experiments that rely on high-precision Doppler measurements.

## DOPPLER TRACKING AS A XYLOPHONE DETECTOR

In Doppler tracking experiments a distant interplanetary spacecraft is monitored from earth through a radio link, and the earth and the spacecraft act as test particles. In a *one-way* operation a radio signal of nominal frequency  $\bar{\nu}_0$  referenced to an on board clock is transmitted to earth, where it is compared to a signal referenced to a highly stable clock. In a *two-way* operation instead a radio signal of frequency  $\nu_0$  is transmitted to the spacecraft, and coherently transponded back to earth. In both configurations relative frequency changes  $\Delta\nu/\nu_0$  as functions of time are measured, and the physical effects the experimenter is trying to observe appear in

the Doppler observable as small frequency changes of well-defined time signature<sup>[3, 4, 5]</sup>.

If a Doppler readout system is added to the spacecraft radio instrumentation, and a transponder is installed at the ground station (Figure 1), one-way as well as two-way Doppler data can also be recorded on board the spacecraft<sup>[1]</sup>. If we assume the earth clock and the on board clock to be synchronized, then the one-way and two-way Doppler data measured at time  $t$  on the earth ( $E_1(t)$ ,  $E_2(t)$  respectively), and the one-way and two-way Doppler measured at the same time  $t$  on the spacecraft ( $S_1(t)$ ,  $S_2(t)$ ), have the following analytic expressions

$$E_1(t) = E_1^0(t) + C_{sc}(t-L) - C_E(t) + T(t) + B(t-L) + A_{sc}(t-L) + EL_{E_1}(t) + P_{E_1}(t) \quad (1)$$

$$E_2(t) = E_2^0(t) + C_E(t-2L) - C_E(t) + 2B(t-L) + T(t-2L) + T(t) + A_E(t-2L) + A_{sc}(t-L) + TR_{sc}(t-L) + EL_{E_2}(t) + P_{E_2}(t) \quad (2)$$

$$S_1(t) = S_1^0(t) + C_E(t-L) - C_{sc}(t) + T(t-L) + B(t) + A_E(t-L) + EL_{S_1}(t) + P_{S_1}(t) \quad (3)$$

$$S_2(t) = S_2^0(t) + C_{sc}(t-2L) - C_{sc}(t) + 2T(t-L) + B(t-2L) + B(t) + A_{sc}(t-2L) + A_E(t-L) + TR_E(t-L) + EL_{S_2}(t) + P_{S_2}(t) \quad (4)$$

where  $E_1^0(t)$ ,  $E_2^0(t)$ ,  $S_1^0(t)$ ,  $S_2^0(t)$  are the contributions of a signal (a gravitational wave pulse or a red shift for instance) to the four Doppler data, and  $L$  is the distance to the spacecraft measured in seconds.

In Equations (1, 2, 3, 4) we have denoted with  $C_E(t)$  the random process associated with the frequency fluctuations of the clock on the earth,  $C_{sc}(t)$  the relative frequency fluctuations of the clock on board,  $B(t)$  the joint effect of the noise from the antenna and buffeting of the spacecraft by non gravitational forces,  $T(t)$  the joint frequency fluctuations due to the troposphere, ionosphere, and ground antenna,  $A_E(t)$  the noise of the radio transmitter on the ground,  $A_{sc}(t)$  the noise of the radio transmitter on board,  $TR_{sc}(t)$  and  $TR_E(t)$  the noise due to the transponder on board and on the ground respectively,  $EL_{E_1}(t)$ ,  $EL_{E_2}(t)$ ,  $EL_{S_1}(t)$ , and  $EL_{S_2}(t)$  the noises from the electronics at the ground station and on the spacecraft in the one-way and two-way data, and  $P_{E_1}(t)$ ,  $P_{E_2}(t)$ ,  $P_{S_1}(t)$ , and  $P_{S_2}(t)$  the frequency fluctuations due to the interplanetary plasma. Note that the noise due to the transmitters on the ground and on board have been denoted with the same random processes ( $A_E(t)$  and  $A_{sc}(t)$  respectively) in the four Doppler responses. This is correct as long as the two radio signals of frequencies  $\nu_0$  and  $\bar{\nu}_0$  transmitted from the earth and the spacecraft are amplified within the operational bandwidth (typically forty to fifty megahertz) of the same transmitters<sup>[13, 14]</sup>. The Doppler data  $S_1(t)$  and  $S_2(t)$  are then time-tagged, and telemetered back to earth in real time or at a later time during the mission.

In Equations (1, 2, 3, 4) it is important to note the characteristic time signatures of the noises  $C_E(t)$ ,  $C_{sc}(t)$ ,  $B(t)$ ,  $T(t)$ ,  $A_E(t)$ , and  $A_{sc}(t)$ <sup>[9, 10, 12]</sup>. It was first pointed out by Vessot

and Levine<sup>[1]</sup> that, by properly combining some of the four Doppler data, it was possible to calibrate the frequency fluctuations of the troposphere, ionosphere, and ground antenna noise,  $T(t)$ . Their pioneering work, however, left open the question on whether there existed some other, perhaps more complicated, linear combinations of the data that would further improve the sensitivity of Doppler tracking. In what follows we answer this question, and derive a method that allows us to uniquely identify an optimal way of combining the data.

Let  $\widetilde{E}_1(f)$  be the Fourier transform of the time series  $E_1(t)$

$$\widetilde{E}_1(f) \equiv \int_{-\infty}^{+\infty} E_1(t) e^{2\pi i f t} dt, \quad (5)$$

and similarly let us denote by  $\widetilde{E}_2(f)$ ,  $\widetilde{S}_1(f)$ ,  $\widetilde{S}_2(f)$  the Fourier transforms of  $E_2(t)$ ,  $S_1(t)$ , and  $S_2(t)$  respectively. The most general linear combination of the four Doppler data given in Eqs. (1, 2, 3, 4), can be written in the Fourier domain as follows:

$$\widetilde{y}(f) \equiv a(f, L) \widetilde{E}_1(f) + b(f, L) \widetilde{E}_2(f) + c(f, L) \widetilde{S}_1(f) + d(f, L) \widetilde{S}_2(f) \quad (6)$$

where the coefficients  $a$ ,  $b$ ,  $c$ ,  $d$  are for the moment arbitrary functions of  $f$  and  $L$ . If we substitute in Equation (6) the Fourier transforms of Eqs. (1, 2, 3, 4) we deduce the following expression

$$\begin{aligned} \widetilde{y}(f) = & [a \widetilde{E}_1^0(f) + b \widetilde{E}_2^0(f) + c \widetilde{S}_1^0(f) + d \widetilde{S}_2^0(f)] + \\ & + \widetilde{C}_E(f) [-a + b(e^{4\pi i f L} - 1) + c e^{2\pi i f L}] + \\ & + \widetilde{C}_{sc}(f) [a e^{2\pi i f L} - c + d(e^{4\pi i f L} - 1)] + \\ & + \widetilde{T}(f) [a + b(e^{4\pi i f L} + 1) + c e^{2\pi i f L} + 2d e^{2\pi i f L}] + \\ & + \widetilde{B}(f) [a e^{2\pi i f L} + 2b e^{2\pi i f L} + c + d(e^{4\pi i f L} + 1)] + \\ & + \widetilde{A}_E(f) [b e^{4\pi i f L} + c e^{2\pi i f L} + d e^{2\pi i f L}] + \\ & + \widetilde{A}_{sc}(f) [a e^{2\pi i f L} + b e^{2\pi i f L} + d e^{4\pi i f L}] + \\ & + a [\widetilde{E} \widetilde{L}_{E_1}(f) + \widetilde{P}_{E_1}(f)] + b [\widetilde{T} \widetilde{R}_{sc}(f) e^{2\pi i f L} + \widetilde{E} \widetilde{L}_{E_2}(f) + \widetilde{P}_{E_2}(f)] + \\ & + c [\widetilde{E} \widetilde{L}_{S_1}(f) + \widetilde{P}_{S_1}(f)] + d [\widetilde{T} \widetilde{R}_E(f) e^{2\pi i f L} + \widetilde{E} \widetilde{L}_{S_2}(f) + \widetilde{P}_{S_2}(f)]. \quad (7) \end{aligned}$$

The four coefficients  $a$ ,  $b$ ,  $c$ ,  $d$ , can be determined by requiring the transfer functions of the random processes  $\widetilde{C}_E(f)$ ,  $\widetilde{C}_{sc}(f)$ ,  $\widetilde{T}(f)$ ,  $\widetilde{B}(f)$ ,  $\widetilde{A}_E(f)$ ,  $\widetilde{A}_{sc}(f)$  in Equation (7) to be simultaneously equal to zero, and by further checking that this solution gives a non-zero signal in the corresponding combined data. This condition implies that  $a$ ,  $b$ ,  $c$ ,  $d$  must satisfy a homogeneous linear system of six equations in four unknowns. We calculated the rank of the  $(6 \times 4)$  matrix associated with this linear system by using the algebraic computer language *Mathematica*, and

we found it to be equal to two. The corresponding solution can be written in the following way

$$\begin{aligned} a(f, L) &= c(f, L) e^{-2\pi i f L} - d(f, L) [e^{2\pi i f L} - e^{-2\pi i f L}] \\ b(f, L) &= -c(f, L) e^{-2\pi i f L} - d(f, L) e^{-2\pi i f L}, \end{aligned} \quad (8)$$

where  $c$  and  $d$  can be any arbitrary complex functions not simultaneously equal to zero. After substituting Equation (8) into Equation (7), we have derived the expressions for the signal in the case of a gravitational wave pulse or in the variation of the gravitational potential as experienced by a spacecraft orbiting a celestial body (redshift measurements). We found that for both these signals their combined Doppler responses (Equation (7)) also vanish. These results imply that, at any Fourier frequency  $f$  and for these two specific Doppler experiments, we can remove only one of the considered noise sources. Among all the noise sources affecting spacecraft Doppler tracking, the frequency fluctuations due to the troposphere, ionosphere, and mechanical vibrations of the ground antenna,  $\tilde{T}(f)$ , are the largest. We can choose  $a$ ,  $b$ ,  $c$ ,  $d$  in such a way that the transfer function of  $\tilde{T}(f)$  in the combined data is equal to zero. From Equation (7) we find that  $a$ ,  $b$ ,  $c$ ,  $d$ , must satisfy the following equation

$$a(f, L) = -b(f, L) [e^{4\pi i f L} + 1] - c(f, L) e^{2\pi i f L} - 2d(f, L) e^{2\pi i f L}. \quad (9)$$

Since  $b$ ,  $c$ ,  $d$  can not be equal to zero simultaneously, we will choose  $c$  to be equal to  $1/2$ , and  $b$ ,  $d$  to be equal to zero. In other words we will consider only linear combinations of one-way Doppler data. Note that with this choice we eliminate from  $y(t)$  the frequency fluctuations due to the transponders and the interplanetary plasma affecting the two-way Doppler data. These considerations imply the following expression for  $\tilde{y}(f)$

$$\begin{aligned} \tilde{y}(f) &= \frac{1}{2} [\tilde{S}_1^0(f) - \tilde{E}_1^0(f) e^{2\pi i f L}] + \tilde{C}_E(f) e^{2\pi i f L} - \frac{1}{2} \tilde{C}_{sc}(f) [e^{4\pi i f L} + 1] + \\ &+ \frac{1}{2} \tilde{B}(f) [1 - e^{4\pi i f L}] + \frac{e^{2\pi i f L}}{2} [\tilde{A}_E(f) - \tilde{A}_{sc}(f) e^{2\pi i f L}] + \\ &+ \frac{1}{2} [\tilde{P}_{S_1}(f) - \tilde{P}_{E_1}(f) e^{2\pi i f L}] + \frac{1}{2} [\tilde{E} \tilde{L}_{S_1}(f) - \tilde{E} \tilde{L}_{E_1}(f) e^{2\pi i f L}]. \end{aligned} \quad (10)$$

Equation (10) shows that the transfer functions of the noise of the on board clock,  $\tilde{C}_{sc}(f)$ , and of buffeting  $\tilde{B}(f)$ , can in principle be set to zero (not simultaneously) at specific Fourier frequencies. In searches for gravitational wave pulses it has been shown<sup>[2]</sup> that one can reduce by several orders of magnitudes the noise of the on-board clock at the nulls of its transfer function without removing the gravitational wave signal. For redshift experiments instead, a cancellation of the noise of the on-board clock at those frequencies removes also the signal. If, however, measurements of  $y(t)$  are made at the Fourier frequencies for which the transfer function of  $\tilde{B}(f)$  is equal to zero, some further improvement in sensitivity can be achieved<sup>[15]</sup>.

## EXPECTED XYLOPHONE SENSITIVITIES

In what follows we provide the expression for the noise in  $y(t)$  at the xylophone frequencies in the case of gravitational wave searches. The analogous expression for redshift experiments can easily be derived in similar fashion.

Let  $\delta$  be the time interval over which a Doppler tracking search for gravitational waves is performed. The corresponding frequency resolution  $\Delta f$  of the data is equal to  $1/\delta$ . This implies that the fluctuations of the clock on board can be minimized at the following frequencies

$$f_k = \frac{(2k-1)}{4L} \pm \frac{\Delta f}{2} ; \quad k = 1, 2, 3, \dots \quad (11)$$

At these frequencies, and to first order in  $\Delta f L$ , the Doppler response  $\tilde{y}(f_k)$  is equal to

$$\begin{aligned} \tilde{y}(f_k) \approx & \frac{1}{2} \left[ \widetilde{S}_1^0(f_k) + i(-1)^k \widetilde{E}_1^0(f_k) \right] + i(-1)^{k+1} \widetilde{C}_E(f_k) \pm (\pi i \Delta f L) \widetilde{C}_{sc}(f_k) + \\ & + \widetilde{B}(f_k) + \frac{1}{2} \left[ \widetilde{E} \widetilde{L}_{S_1}(f_k) - i \widetilde{E} \widetilde{L}_{E_1}(f_k) (-1)^{k+1} \right] + \\ & + \frac{1}{2} \left[ \widetilde{P}_{S_1}(f_k) - i \widetilde{P}_{E_1}(f_k) (-1)^{k+1} \right] + \\ & + \frac{1}{2} \left[ \widetilde{A}_{sc}(f_k) + i \widetilde{A}_E(f_k) (-1)^{k+1} \right] . \end{aligned} \quad (12)$$

For a typical gravitational wave experiment,  $\delta = 40$  days, and  $\Delta f = 3.0 \times 10^{-7}$ . Therefore, the frequency fluctuations of a clock on board a spacecraft that is out to 1 AU are reduced at the xylophone frequencies by the following amount:

$$\frac{\pi \Delta f L}{c} = 4.7 \times 10^{-4} .$$

We should point out, however, that these resonant frequencies in general will not be constant, since the distance to the spacecraft will change over a time interval of forty days. As an example, however, let us assume again  $L = 1$  AU,  $\delta = 40$  days, and  $f = 5 \times 10^{-4}$  Hz. The variation in spacecraft distance corresponding to a frequency change equal to the resolution bin width ( $3 \times 10^{-7}$  Hz) is equal to  $1.0 \times 10^5$  Km. Trajectory configurations fulfilling a requirement compatible to the one just derived have been observed during past spacecraft missions<sup>[9]</sup>, and therefore we do not expect this to be a limiting factor.

From Equation (12) we can estimate the expected root-mean-squared (r.m.s.) noise level  $\sigma(f_k)$  of the frequency fluctuations in the bins of width  $\Delta f$ , around the frequencies  $f_k$  ( $k = 1, 2, 3, \dots$ ). This is given by the following expression

$$\sigma(f_k) = [S_y(f_k) \Delta f]^{1/2} , \quad k = 1, 2, 3, \dots , \quad (13)$$

where  $S_y(f_k)$  is the one-sided power spectral density of the noise sources in the Doppler response  $y(t)$  at the frequency  $f_k$ . In what follows we will assume that the random processes representing these noises are uncorrelated with each other, and their one-sided power spectral densities are as given in Table I. In this table we have assumed a frequency stability of  $1.0 \times 10^{-16}$  at 1000 seconds integration time for the clock at the ground station. Although this is a factor of four better than what has been measured so far<sup>[11]</sup>, it seems very likely that by the beginning of next century such a sensitivity can be achieved. As far as the other sensitivity figures provided in Table I are concerned, they are as given in the Riley *et al.* report<sup>[11]</sup>. This document is a summary of a detailed study, performed jointly by scientists and engineers of NASA's Jet Propulsion Laboratory and the Italian Space Agency (ASI) Alenia Spazio, for assessing the magnitude and spectral characteristics of the noise sources that will determine the Doppler sensitivity of the future gravitational wave experiment on the Cassini mission. Included in Table I is also the spectral density of the noise of a crystal-driven ultra-stable oscillator (USO).

If dual radio frequencies in the uplink and downlink are used, then the frequency fluctuations due to the interplanetary plasma can be entirely removed<sup>[11]</sup>. We will refer to this configuration as MODE I. If only one frequency is adopted instead, which we will assume to be Ka-Band (32 GHz), we will refer to this configuration as MODE II. Ka-Band is planned to be used on most of the forthcoming NASA missions, and will be implemented on the ground antennas of the Deep Space Network (DSN) by the year 1999 for the Cassini mission.

In Figure 2 we plot the r.m.s.  $\sigma(f_k)$  of the noise as a function of the frequencies  $f_k$  ( $k = 1, 2, 3, \dots$ ), assuming that an interplanetary spacecraft is out to a distance  $L = 1.0$  AU. For this configuration the fundamental frequency of the xylophone (Equation (11)) is equal to  $5.0 \times 10^{-4}$  Hz. A complete analysis covering configurations with spacecraft at several other distances is given in<sup>[2]</sup>.

The MODE I configuration is represented by two curves, depending on whether an atomic clock (circles) or a USO (squares) is operated on board the spacecraft. Sensitivity curves for the MODE II configuration are also provided, again with an atomic clock on board (up-triangles) or a USO (down-triangles). The best sensitivity is achieved in the MODE I configuration, regardless of whether an atomic clock or a USO is operated on board the spacecraft (circles and squares are over imposed). This is because the amplitude of the noise of the clock on board is reduced by a factor  $\pi \Delta f L / c = 4.7 \times 10^{-4}$  at the xylophone frequencies. At  $f = 10^{-3}$  Hz the corresponding r.m.s. noise level is equal to  $4.7 \times 10^{-18}$ , and it increases to a value of  $5.7 \times 10^{-18}$  at  $f = 10^{-2}$  Hz. As far as the MODE II configuration is concerned, the r.m.s. noise level is equal to  $7.9 \times 10^{-18}$  at  $f = 10^{-3}$  Hz, while it decreases to  $6.3 \times 10^{-18}$  at  $f = 10^{-2}$  Hz. This is due to the fact that the one-sided power spectral density of the fractional frequency fluctuations due to the interplanetary plasma decays as  $f^{-2/3}$ .

In Figure 3 we turn to redshift experiments with a spacecraft out to 5 AU, and we assume an observing time of 40 hours. This example can be considered as representative of a spacecraft orbiting the planet Jupiter. The reduction factor of the buffeting noise  $B(t)$  (see Equation (10)) is now equal to

$$\frac{\pi \Delta f L}{c} = 5.5 \times 10^{-2}, \quad (14)$$



and the xylophone frequencies are given by the following relation

$$f_k = \frac{k}{2L} \pm \frac{\Delta f}{2} ; \quad k = 1, 2, 3, \dots \quad (15)$$

We also have assumed that when plasma calibration is not implemented, the frequency fluctuations due to interplanetary scintillation are estimated at opposition. This of course does not represent a general situation but only an example.

The best sensitivity is achieved in the MODE I configuration and by using an atomic clock on board. At the Fourier frequency  $f = 2.0 \times 10^{-4}$  Hz the sensitivity is equal to  $3.0 \times 10^{-17}$ , and it increases slowly at higher frequencies. In the MODE II configuration and with an atomic clock on board the sensitivity degrades by about a factor of three with respect to the previous configuration. If a USO is used instead, then MODE I and II are totally equivalent, since the USO is the dominant noise source. In this case at  $f = 2.0 \times 10^{-4}$  the sensitivity of the xylophone is equal to  $1.4 \times 10^{-14}$ , and improves as  $f^{-1/2}$  as the frequency increases.

## CONCLUSIONS

We have discussed a method for significantly increasing the sensitivity of two Doppler tracking experiments, namely searches for gravitational waves and measurements of the redshift effect. Our method relies on a properly chosen linear combination of the one-way Doppler data recorded on board with those measured on the ground. It allows us to remove entirely the frequency fluctuations due to the troposphere, ionosphere, and antenna mechanical, and for a spacecraft that is tracked for forty days out to 1 AU in search for gravitational waves, it reduces by almost four orders of magnitude the noise due to the on-board clock. For a redshift experiment instead, with a spacecraft out to 5 AU, our technique allows us to reduce by about two orders of magnitude the noise of the antenna and buffeting of the spacecraft.

The experimental technique presented in this paper can be extended to a configuration with two spacecraft tracking each other through a microwave or a laser link. Future space-based laser interferometric detectors of gravitational waves<sup>[16]</sup>, for instance, could implement this technique as a backup option, if failure of some of their components would make the normal interferometric operation impossible.

As a final note, a method similar to the one presented can be used in all those radio science experiments in which one-way and two-way spacecraft Doppler measurements are used as primary data set. We will analyze the implications of the sensitivity improvements that this technique will provide for direct measurements of the following quantities such as searches for possible anisotropy in the velocity of light, measurements of the Parameterized Post-Newtonian parameters, measurements of the deflection and time delay by the sun in radio signals, and occultation experiments. This research is in phase of development, and will be the subject of a forthcoming paper.

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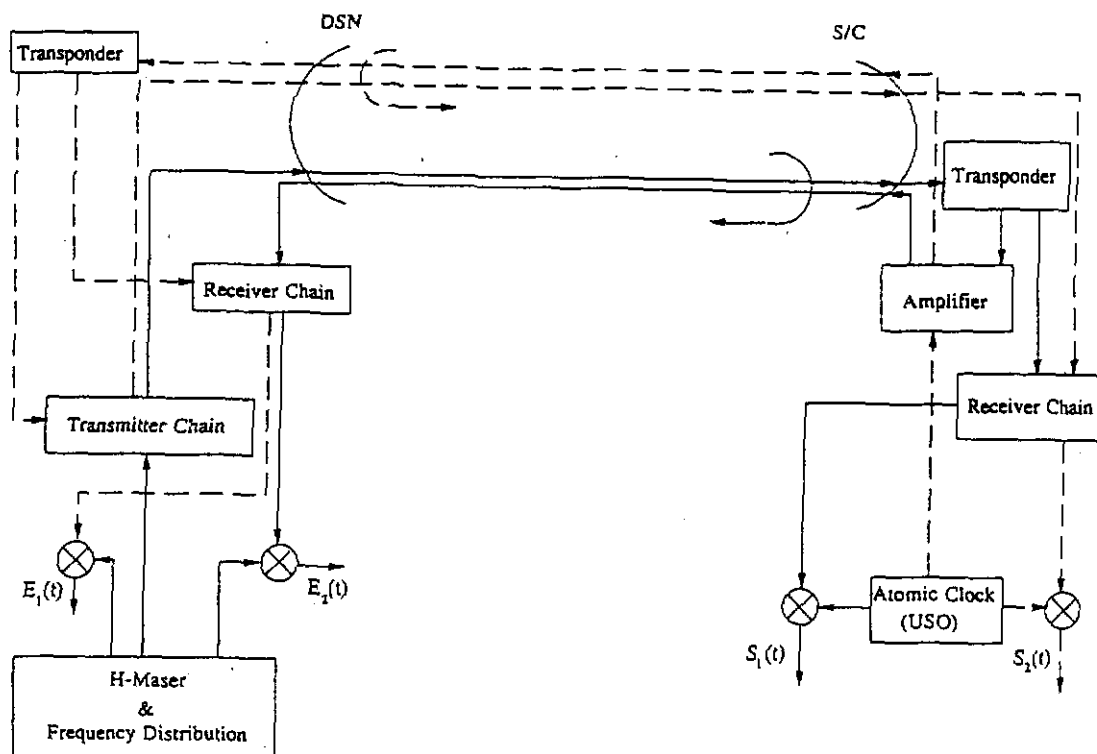


Figure 1.

Block diagram of the radio hardware at the ground antenna of the NASA Deep Space Network (DSN) and on board the spacecraft (S/C), that allows the acquisition and recording of the four Doppler data  $E_1(t)$ ,  $E_2(t)$ ,  $S_1(t)$ , and  $S_2(t)$ .

Error Source	Allan Deviation @ 1,000 sec.	Fractional Frequency One-Sided Power Spectral Density
One-Way Plasma (@ Ka-Band)	$4.9 \times 10^{-16}$	$2.7 \times 10^{-10} f^{-2/3}$
H-Maser	$1.0 \times 10^{-16}$	$6.2 \times (10^{-28} f + 10^{-33} f^1 + 10^{-30})$
Frequency Distribution	$1.0 \times 10^{-17}$	$1.3 \times 10^{-26} f^2$
Receiver chain	$3.1 \times 10^{-17}$	$1.3 \times 10^{-25} f^2$
Transmitter chain	$3.4 \times 10^{-16}$	$2.3 \times 10^{-28}$
Thermal Noise	$3.8 \times 10^{-17}$	$1.9 \times 10^{-25} f^2$
Spacecraft Antenna & Buffeting	$5.8 \times 10^{-17}$	$5.0 \times 10^{-42} f^{-3} + 10^{-31} + 5.0 \times 10^{-30}$
Spacecraft Amplifier	$5.0 \times 10^{-17}$	$4.0 \times 10^{-27} f$
USO	$9.5 \times 10^{-14}$	$6.5 \times 10^{-27} f^{-1}$

Table I

List of the noise sources entering into the combined Doppler response  $y(t)$ . The Allan deviation at a given integration time  $\tau$  is a statistical parameter for describing frequency stability. It represents the root-mean-squared expectation value of the random process associated with the fractional frequency changes, between time-contiguous frequency measurements, each made over time intervals of duration  $\tau$ . The numbers provided in this table are taken from the Riley et al. report [11].

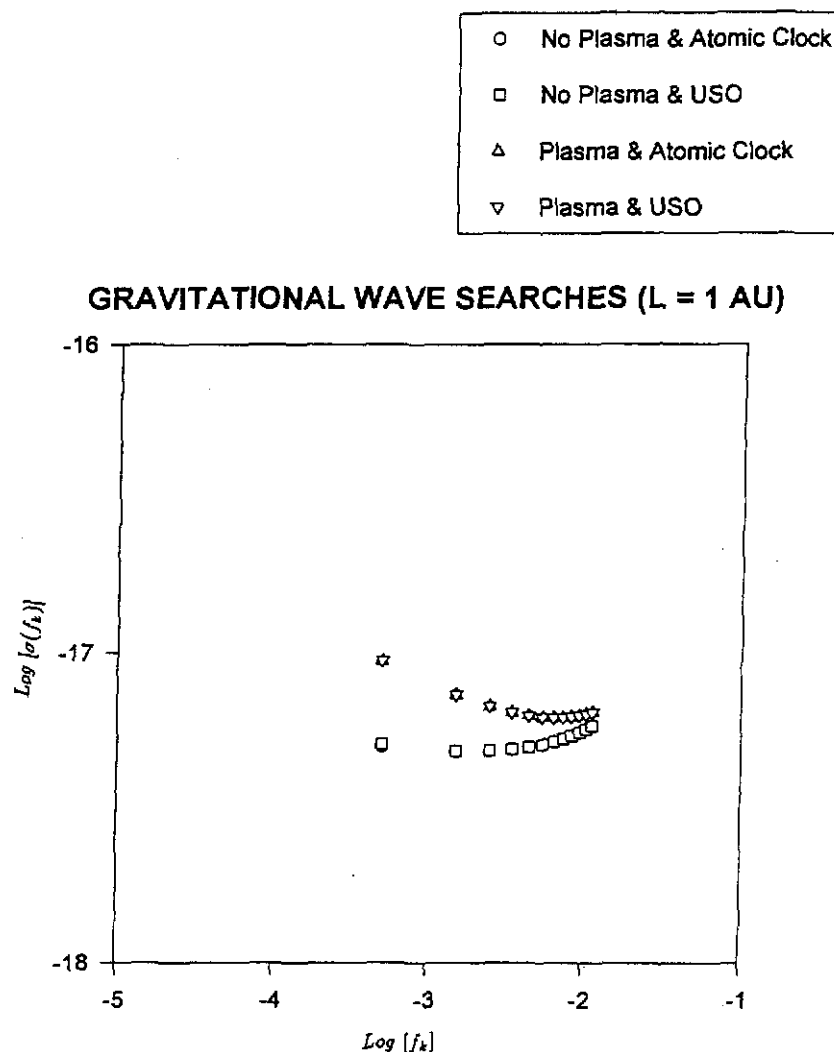
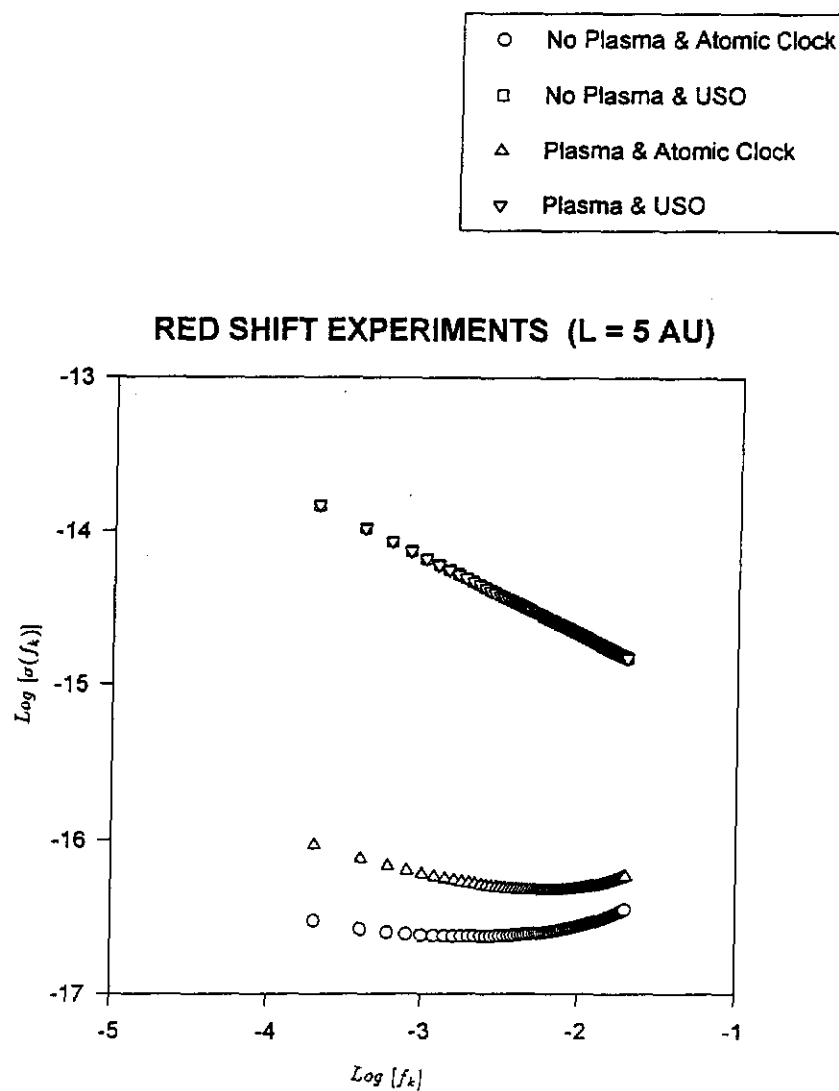


Figure 2.

The r.m.s. noise level as a function of the frequencies  $f_k$  ( $k = 1, 2, 3, \dots$ ), estimated for a spacecraft that is out to a distance  $L = 1.0 \text{ AU}$  searching for gravitational waves. Sensitivity figures for the four distinct configurations are represented with four different symbols. Circles represent r.m.s. values as functions of the xylophone frequency when plasma frequency fluctuations are totally removed and an atomic standard is on board. Squares are sensitivity values again with plasma calibration, but now with a USO on the spacecraft. Up-triangles are used for representing sensitivity figures affected by plasma noise at Ka-Band, and an atomic clock is used on board. Down-triangles assume a plasma noise at Ka-Band, but now with a USO on board.



**Figure 3.**

The r.m.s. noise level as a function of the frequencies  $f_k$  ( $k = 1, 2, 3, \dots$ ), estimated for a spacecraft that is out to a distance  $L = 5.0$  AU and tracked during a red shift experiment.